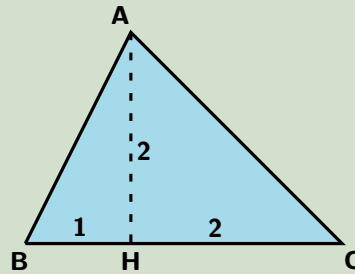


## February 2018 Detroit Mercy Math Problem

### Special $m:n$ Triangles

An acute angled triangle  $ABC$  is called a *special  $m:n$  triangle with vertex  $A$*  if  $AH : BH : CH$  are in the proportion  $mn : m : n$ , where  $H$  is the foot of the altitude dropped from the vertex  $A$  on to the base  $BC$ .

Here is a special  $1:2$  triangle, with  $AH : BH : CH = 2 : 1 : 2$ .



And here is the February 2018 problem:

### Problem

In the figure below,  $ABCDE$  is a pentagon such that (with respect to vertex  $A$ ):

1.  $ABC$  is a special  $3:5$  triangle,
2.  $ACD$  is a special  $2:7$  triangle, and
3.  $ADE$  is a special  $8:3$  triangle.

Prove that the pentagon's vertex angle  $\angle A$ , that is  $\angle BAE$ , must be a right angle, *using only elementary geometry/trigonometry, and without using a calculator or a computer.*

